A Stochastic Model of Knee Angle in Response to Electrical Stimulation of the Quadriceps and Hamstrings Muscles

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SHORT TITLE – Stochastic Model of Knee Angle in Response to Electrical Stimulation

ABSTRACT

A novel stochastic model of knee angle in response to stimulation of the quadriceps and hamstrings muscle groups is presented. This model includes uncertainty due to fatigue and day-to-day changes in the stimulated muscles. The model consists of a normally distributed random variable whose mean and standard deviation vary with time, and is characterized using data from a complete SCI subject. The experimental data show a significant difference between the left and right legs under certain conditions, and suggest that fatigue-related and day-to-day variation may also be important. The purpose of this model is to generate more realistic electrically stimulated knee movements. This stochastic modeling technique could be incorporated into a comprehensive model of a joint actuated with electrical stimulation, and has great potential as a tool for analyzing closed-loop performance of electrically stimulated systems.

KEYWORDS: model, stochastic, knee, functional electrical stimulation, spinal cord injury

I. INTRODUCTION

Functional electrical stimulation (FES) uses short electrical pulses to generate contractions in paralyzed muscles. FES can be used to restore or replace motor function in

individuals with spinal cord injuries (SCI) by coordinating the elicited contractions to move or stabilize the joints affected by the SCI. The strength of the contractions is controlled by modulating the intensity of stimulation delivered to the muscles. FES can be used for many applications in individuals with SCI. FES-based rehabilitation modalities include gait training [1] and upper limb rehabilitation [2, 3]. FES neuroprostheses that replace lost motor function include systems for grasping [4, 5], elbow extension [6], standing [7-9], walking [10, 11], trunk stabilization [12, 13], and improving orthostatic tolerance [14, 15].

The response of muscles to electrical stimulation is nonlinear, time-varying, coupled to the behavior of other muscles, and is also subject to strong disturbances [16]. The FES community is presently lacking a model that captures the unpredictable nature of the stimulated muscle response. Such a model could greatly facilitate the development of sophisticated new FES systems by allowing the designers to test control algorithms under realistic conditions prior to testing with SCI subjects.

The first objective of the work presented here is to verify that the anecdotally observed variation in electrically induced knee movement is indeed statistically significant. The second objective is to develop a model of electrically stimulated knee movement that incorporates this variation. This article concerns changes in knee angle due to quadriceps and hamstrings stimulation, however these results could be generalized to other joints.

Several models of knee angle due to quadriceps and hamstrings stimulation are available. Hatwell et al. present a knee model based on a deterministic autoregressive moving average model of the leg dynamics in [17]. Previdi presents a nonlinear autoregressive exogenous model of knee movement as a result of quadriceps stimulation in [18]. Ferrarin and Pedotti use a nonlinear second-order system to model the dynamics of the knee and lower leg, and a one-pole

transfer function to model the relationship between stimulation pulse width and quadriceps torque [19]. Perumal, Wexler, and Binder-MacLeod propose a model of knee angle in response to a various quadriceps stimulation parameters, and test the model with healthy subjects [20]. These models provide detailed, deterministic information, but are not designed to capture the large amount of day-to-day and fatigue-related variations in electrically stimulated knee angle that is seen in individuals with SCI.

A few models of electrically stimulated knee angle do include some sources of variation. Schauer et al. propose a model that uses a time-varying component to describe day-to-day variation in the stimulated knee angle [21]. Riener proposes a knee model that includes intra-session variation due to fatigue [22]. To our knowledge, no model of electrically stimulated knee angle is available that includes all sources of day-to-day and intra-session variations.

We develop a stochastic model of electrically stimulated knee angle that captures both day-to-day and fatigue-related variation. The information provided by this model could be combined with traditional knee models, such as those reviewed above. The integrated models could then be used to generate a more realistic estimate of knee angle in response to electrical stimulation. This type of model will allow engineers to develop FES systems that are more robust and able to compensate for day-to-day and intra-session variability in the electrically stimulated muscle response observed in individuals with SCI.

II. METHODS

II-A. Experimental Protocol

One subject with a complete SCI (AIS A) at level T3 took part in this study, which was approved by the local research ethics board. The data collected for this study were part of a larger planned study with 10 subjects. However, 9 of the 10 subjects withdrew from the study

before the data collection phase was completed for various reasons, i.e. pregnancy, found employment, returned to school, sustained an injury unrelated to the study, and recurrence of health problems such as pressure ulcers and urinary tract infections. The remaining subject did not have any known differences between the right and left legs, and did not report any spasticity. The subject participated in a FES-based muscle strengthening protocol 3 days per week for 8 weeks prior to the data collection phase of the project. During each hour-long strengthening session, the subject was seated with the shank free to swing. An electrogoniometer (Biopac Systems Inc., USA) recorded the knee angle, and was connected to a NI 6040E data acquisition card (National Instruments, USA) through an electrical isolator. Custom software sampled the knee angle at 200 Hz and controlled the Compex Motion stimulator unit (Compex SA, Switzerland).

Adhesive 5 cm square electrodes delivered the stimulation to the subject's quadriceps and hamstrings muscles; one electrode was affixed to the motor point of each muscle, and a return-path electrode was affixed distal to the first electrode on each muscle group. For this particular subject, stimulating only the m. vastus lateralis of the quadriceps provided the most anatomically correct knee movement. The vastus medialis and rectus femoris were not targeted for stimulation, because these muscles caused knee adduction and were significantly atrophied, respectively. The subject's muscles were repeatedly stimulated with a train of biphasic rectangular pulses (pulse width 250 μs, frequency 40 Hz) for 5 s to allow the knee angle to reach steady state, followed by a 5 s rest period of no stimulation, during which time the knee returned to the rest position. The amplitude for each muscle was set to a pre-defined level (quadriceps: 95 mA, hamstrings: 65 mA) that elicited the maximal range of motion from the knee. The subject's left and right quadriceps and hamstrings muscle groups were exercised sequentially.

A similar protocol was followed during the data collection phase, however only the right or left leg was used in a particular session. Also, the quadriceps and hamstrings muscles were stimulated simultaneously during data collection. Finally, the stimulation amplitude for each of the two muscle groups was randomly selected by the computer from a discrete set of values between 0 mA and the pre-set maximum amplitude.

A trial consisted of 5 s of stimulation followed by 5 s of rest. The first 0.05 s of each trial was disregarded to exclude any initial transient noise. The data was also processed to eliminate any obviously spurious data points, which were defined as any data points for which the angular velocity of the knee exceeded a pre-set threshold. This threshold was set to 400 degrees per second, which was the mean knee angular velocity seen during fast walking in able-bodied individuals [23]. These spurious data points were replaced with the mean of the previous three samples to approximate the local behavior of the knee angle. The processed data was smoothed using a 5-point moving average to reduce sensor noise in the data. Each trial was also translated to the same starting knee angle to eliminate the slight trial-to-trial variation in resting knee angle.

II-B. Analysis of Variation in Experimental Data

The data were analyzed to examine the significance of the variations in the electrically induced knee movements. The trials were grouped by stimulation level, and then subdivided into left early, left middle, left late, right early, right middle, and right late trials. There is little published data on the rates of the fatigue and recovery in electrically stimulated muscle in SCI individuals, so trials 1 to 30 were arbitrarily defined as early trials, trials 31 to 60 as middle trials, and trials 61 to 90 as late trials.

Table 1 lists the statistical tests that were performed. The tests were conducted for each set of stimulation parameters for which there were at least two trials in each group A, B, and C (where applicable). The metric used was the root-mean-squared error between a trial and the mean of the population of trials in the test, across groups A, B, and C, and $\alpha = 0.05$ was used.

II-C. Modeling Methods

A stochastic model of knee angle as a function of quadriceps and hamstrings stimulation, as well as time, was constructed, and was characterized using the experimental data. For a particular set of stimulation conditions, the knee angle $y(t)$ was described using a model of the form $y(t) = Y(t; a_1, \dots, a_M)$, where $Y(t; a_1, \dots, a_M)$ was a normally distributed random variable having mean $\mu(t;a_1,...,a_M)$ and standard deviation $\sigma(t)$. The mean $\mu(t;a_1,...,a_M)$ was defined to be a linear combination of nonlinear basis functions:

$$\mu(t; a_1, \dots, a_M) = \sum_{i=1}^{M} a_i g_i(t). \qquad (1)$$

The number of basis functions M and the parameters $a_1, ..., a_M$ were chosen to yield the best-fit model to the experimental data. The basis functions were hand-selected to represent aspects of the electrically induced knee movements that were observed across all experimental sessions:

$$g_i(t) = \begin{cases} t^{i-1}, & i = 1, 2, \dots, 5 \\ \exp(-\beta_i t)\sin(2\pi\omega_i t) & i = 6, 7, \dots, M \end{cases}$$

where $\beta = \{1, 1.5, 2, 2.5, 3\}$ and $\omega = \{1.12, 1.15, 1.18\}$. The standard deviation $\sigma(t)$ of the random variable $Y(t;a_1,...,a_M)$ was defined to be equal to the standard deviation of the data from the best-fit model in (1), for a particular set of stimulation conditions.

A linear least squares method was used to find the coefficients $a_1, ..., a_M$ of the best-fit model with M basis functions [24]. Using this method, a chi-square statistic was defined to be:

$$\chi^2 = \sum_{j=1}^{N} \left(\frac{y_j - \sum_{i=1}^{M} a_i g_i(t_j)}{\sigma(t_j)} \right)^2,$$

where y_j is the mean of the experimental data at time t_j.

Next, a goodness-of-fit metric $Q(\chi^2|\nu)$ was defined, which indicated the likelihood of a particular best-fit model with M basis functions being the "correct" model for a particular set of stimulation conditions. To use the probability distribution $Q(\chi^2|\nu)$ as a goodness-of-fit metric, the chi-square statistic χ_0^2 was calculated for the experimental data and a particular best-fit model with M basis functions. Next, the value of $Q(\chi_0^2|\nu)$ was found by computing the upper incomplete Gamma function with first parameter 0.5 ν, ν = N-M, and second parameter 0.5 χ_0^2 [25]. A high value for $Q(\chi_0^2|\nu)$ meant that the best-fit model was likely to be the correct model, of which the experimental data was a sample.

To find the stochastic model for each set of stimulation conditions using the modeling procedure outlined above, the mean y(t) was calculated for each group of trials $r_k(t)$, k = 1,..., d, where a particular experimental trial was denoted $r_i(t)$, and d was the number of trials in the group. Then, the best-fit model with M basis functions that minimized the chi-square statistic was found, for M = 1,..., 20. Next, the best-fit model with the most favorable $Q(\chi^2|\nu)$ value was chosen as the correct model. This correct model became the mean $\mu(t; a_1,...,a_M)$ of the random variable $Y(t; a_1,...,a_M)$, and the standard deviation of the random variable $Y(t; a_1,...,a_M)$ was defined to be the standard deviation of the trials $r_k(t)$ from the correct model.

III. RESULTS

III-A. Statistical Analysis of Variation

Table 2 reports some examples of stimulation conditions for which there was a statistically significant difference between the left and right leg trials. However, not all

stimulation parameters were associated with a sufficient number of trials to conduct the analysis. A similar phenomenon was seen when analyzing fatigue-related variation. The stimulation parameters that were associated with a sufficient number of trials to conduct the analysis showed that, in some cases, the variation between early, middle, and late trials approached but did not reach significance. For day-to-day variation, a significant difference could not be shown between trials conducted under the same conditions but on different days.

III-B. Stochastic Models of Knee Angle in Response to Quadriceps and Hamstrings Stimulation

Table 3 lists some representative numerical results for stochastic models of stimulated knee movements for early left leg trials versus early right leg trials versus early trials on either leg. The number of basis functions in the correct model, the goodness-of-fit $Q(\chi^2|\nu)$ for the correct model, and the pooled standard deviation of the experimental data from the correct model are reported. Figure 1 shows the modeling results for all early trials conducted at 86 mA quadriceps and 0 mA hamstrings stimulation, comparing the differences between the left and right legs.

IV. DISCUSSION

For certain sets of stimulation conditions, a statistically significant difference exists between the angle of the right versus left knees in response to electrical stimulation of the quadriceps and hamstrings. However, this difference cannot be shown for all sets of stimulation conditions. The modeling results suggest that a difference does exist between the electrically induced movements of the left and right knees. For example, Figure 1a shows that the coefficients of these particular left and right models are different. Moreover, the modeling results also suggest that day-to-day and fatigue-related differences exist.

Figures 1b-1d show that the stochastic modeling method generates a model that represents the range of experimentally observed behavior, as illustrated by the envelopes of maximum and minimum knee angle over 100 instances of the stochastic best-fit model encompassing the experimental data. This stochastic model could be generalized to arbitrary constant stimulation parameters by interpolating between the model coefficients for the constant stimulation case. The model could also be applied to other joints or muscles, or generalized to include other parameters such as different initial knee angles or dynamic stimulation parameters, provided that an appropriate data set is available. Additional basis functions may have to be incorporated into the stochastic model to allow it to represent an expanded set of conditions.

The limitations of this study include the single-subject nature of the experiment, as well as the limited number of experimental sessions. It is likely that stronger evidence of significant differences would have been found if it had been possible to collect data from the other subjects who were recruited for the study. Sample size was not calculated a priori, because there was an insufficient amount of data on the variability of the response of stimulated muscle in the literature to support such calculations.

Although sources of error were controlled as much as possible, the differences seen between the response of the left versus right legs could be due to sensor error or variation in stimulation levels. It should also be noted that the response of electrically stimulated muscle is not actually a stochastic process. However, it is not currently possible to measure all the internal variables that determine exactly how a muscle will respond to electrical stimulation, so the variations in muscle response appear random.

V. CONCLUSIONS

The results presented in this article show that the intra-subject variation between the electrically induced knee movement of the left and right legs can be significant, and suggest that day-to-day and fatigue-related variation are also important. Since this variation is unlikely to be an isolated case, it would be prudent to account for variability in the response of electrically stimulated muscles when developing and testing FES systems for use by SCI individuals.

The novel stochastic model presented in this article represents the range of behavior that can be expected of the knee for a particular set of constant quadriceps and hamstrings stimulation conditions, assuming a fixed initial knee angle, and includes day-to-day, inter-limb, and fatigue-related intra-session variation. This type of model could be used to augment the currently available deterministic models used in FES systems. The resulting comprehensive model would describe the typical response of the system to muscle stimulation as well as the variation in the response that can be expected. Such a comprehensive model could be used in model-based control algorithms or to develop realistic simulations for verifying the performance of FES systems, thereby economizing the time and resources required for final human subject testing.

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TABLES & FIGURES

Table 1 – Statistical tests performed. X, Y, and Z refer to particular experimental sessions.

Null Hypothesis	Test	Trials in Group A	Trials in Group B	Trials in Group C
No left-right difference	t-test	Left leg	Right leg	N/A
No fatigue-related diff.	ANOVA	Early, left leg	Middle, left leg	Late, left leg
		Early, right leg	Middle, right leg	Late, right leg
No day-to-day difference	ANOVA	Early, left leg, day X	Early, left leg, day Y	Early, left leg, day Z
		Early, right, day X	Early, right, day Y	Early, right, day Z

Table 2 – Examples of statistical analysis of variation between left and right leg trials. In each case, the hamstrings stimulation was 0 mA. ** denotes statistical significance ($p < 0.01$).

Quadriceps Stimulation	t ratio	Critical Value of t ($p < 0.01$)	Effect Size (d)
81 mA	$t(14) = 6.25$**	2.98	3.15
86 mA	$t(17) = 5.06$**	2.90	2.33
90 mA	$t(12) = 12.78$**	3.06	6.83

Table 3 – Numerical results for stochastic models of early left leg trials versus early right leg trials versus early trials on either leg, for 86 mA quadriceps and 0 mA hamstrings stimulation. Pooled standard deviation refers to the deviation of the experimental data from correct model.

	Both Legs	Left Leg	Right Leg
Number of Basis Functions in Correct Model	6	7	6
Goodness-of-Fit of Correct Model	0.812	0.956	0.999
Pooled Standard Deviation	10.075	8.036	10.956

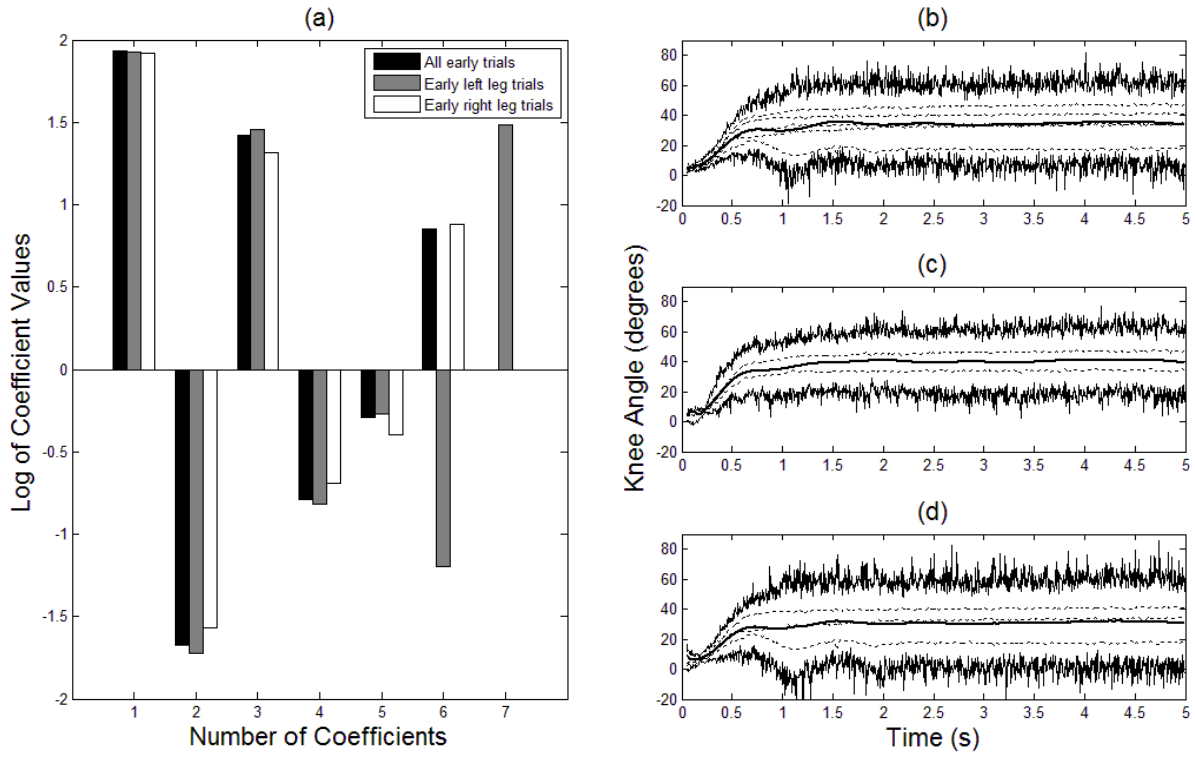

Figure 1 – Stochastic best-fit models of early trials at 86 mA quadriceps and 0 mA hamstrings stimulation. (a) Model coefficients, showing variation between left and right legs. (b) Experimental data and model results for trials with either left or right leg. Heavy solid line is expected value of stochastic best-fit model. Dashed lines are experimental data. Thin solid lines define maximum (upper line) and minimum (lower line) values of 100 instances of stochastic best-fit model. (c) Left leg trials only. (d) Right leg trials only.